MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2019/2020

EME3156 – COMPUTATIONAL METHODS FOR MECHANICAL ENGINEERING

(ME)

04 MARCH 2020
9.00 a.m. - 11.00 a.m.
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This Question paper consists of 8 pages with 4 Questions only, excluding the cover page.

2. Attempt **ALL** questions. All questions carry equal marks and the distribution of the marks for each question is given.

3. Please write all your answers in the Answer Booklet provided.

4. Appendices are attached at Page 7 and 8.

Question 1

Equation (1.1) shows a first order ODE and the initial condition.

$$\frac{dy}{dx} = 2x^2, \; y(0) = 3 \tag{1.1}$$

(a) Verify that the analytical solution of the above equation is: $y = \frac{2}{3}x^3 + 3$.

[5 marks]

(b) Find $y(1)$ by using the explicit Euler method with step size $h = 0.25$,

[10 marks]

(c) Make a sketch to show the concept of the midpoint method. The initial condition and approximated values must be labelled and shown clearly.

[4 marks]

(d) Find $y(1)$ by using the midpoint method with step size $h = 1$,

[4 marks]

(e) Find the exact value of $y(1)$. Then, based on the answer in (b) and (d), state which method gives the closer answer to the exact value.

[2 marks]

Continued...

Question 2

Equation (2.1) shows a non-linear second order ODE and the boundary conditions. The domain is discretised into six nodes equally distanced and it is to be solved using finite difference method.

$$\frac{d^2y}{dx^2} - 2y^2 = 7x,\ y(0) = 2,\ y(1) = 2 \tag{2.1}$$

(a) Sketch the discretisation with the node numbering in sequence starting from "0" at the left boundary.

[3 marks]

(b) Write down the difference form of Equation (2.1) in numerical values as much as possible by using the central difference for all terms, and then apply this equation to the discretised domain. The boundary conditions must be incorporated into these equations.

[12 marks]

(c) The Newton method for non-linear system of equations is used to solve the equations formed in (b), where these equations are assembled to become a matrix equation $[J]\{\Delta y\} = \{F\}$. Expand this matrix equation to show all elements in terms of the nodes in the discretisation. Appropriate subscripts that are consistent with the node numbering must be used.

[10 marks]

Continued...

Question 3

There is a model of one-dimensional heat conduction through a thin layer of insulation. It has four nodes, numbered 1, 2, 3 and 4.

The boundary conditions (BC's) occur at nodes 1 and 4. The BC's are:

$$T_1 = 25°C \qquad T_4 = -5°C$$

The thermal diffusivity is α, measured in units of m^2/s.

$$\alpha = 10 \times 10^{-8}\ m^2/s$$

Let r be a factor in the temperature distribution across the nodes. The equation for r is:

$$r = \alpha \frac{\Delta t}{(\Delta y)^2} \quad , \quad 0 < r < 0.5$$

Δt is the amount of time passed during one step, i.e. from t_i to t_{i+1}.
Δy is the distance between each node.

$$\Delta y = 5 \times 10^{-3}\ m$$

The equation that is used on the computation of the model with finite difference is Equation (3.1):

$$T_i^{t+1} - T_i^t = r(T_{i-1}^t - 2T_i^t + T_{i+1}^t) \tag{3.1}$$

To apply numerical computation to Equation (3.1), it is to be converted into the matrix Equation (3.2):

$$\{T\}^{t+1} = [r]\{T\}^t + \{BC\} \tag{3.2}$$

$[r]$ is the matrix that contains statements of r, separated from the temperature variables.

The initial conditions of nodes 2 and 3 at $t = 0$ are: $T_2^0 = 10°C$ and $T_3^0 = 10°C$

(a) Sketch a diagram for the model.

[4 marks]

(b) The factor of r has to be as close as possible to 0.5, but Δt must be exact multiples of 100. Find the biggest value of Δt in units of seconds.

[2 marks]

(c) Using the answer for part (b), find the value of r.

[2 marks]

(d) Briefly describe one advantage and one disadvantage of using a large time step Δt for the numerical computation of this model.

[4 marks]

Continued...

(e) Apply Equation (3.1) on the model to obtain equations that include T_1, T_2, T_3 and T_4. (Hint: There should be two equations.)

[2 marks]

(f) Rearrange the equations from part (e) into the matrix form as shown in Equation (3.2).

[4 marks]

(g) Solve the matrix equation from part (f) for times $t = 0, 1, 2$ and so on until T_2 and T_3 reach a stable temperature. Mention the stable temperature.
(The temperature can be considered stable if the next time step gives results that have differences from the previous step at amounts of 0.1°C or less.)

[5 marks]

(h) From the answers from parts (b) and (g), calculate the actual amount of time in seconds to reach stable temperature.

[2 marks]

Continued...

Question 4

There is to be a 2D model of the cross section of a heating element. The model is to be composed of nodes arranged in quadrilaterals. Figure 4(a) is a reference to this arrangement. The model has a source of heat generation in its center, as shown in Figure 4(b). The model is to have nine nodes.

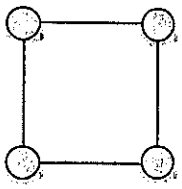

Figure 4(a)

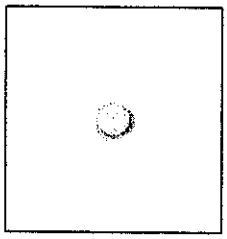

Figure 4(b)

The distance between one node and the next is $\Delta x = \Delta y = 1 \times 10^{-3}$ m.

Node 5 is to be the node that represents the source of heat generation. The source is generating heat at the following rate:

$$\dot{Q}_5(\Delta t) = 5°C/s$$

The other nodes are maintained at specific temperatures, which are as follows:

$$T_1 = T_2 = T_3 = T_4 = T_6 = T_7 = T_8 = T_9 = 50°C$$

The temperature change equation for node 5 is:

$$\frac{\partial T_5}{\partial t} = \alpha \frac{\partial^2 T_5}{\partial x^2} + \alpha \frac{\partial^2 T_5}{\partial y^2} + \dot{Q}_5$$

The initial temperature for node 5 is 50°C.

Let r be a factor in the temperature change, with the following equation and value:

$$r = \alpha \frac{(\Delta t)}{(\Delta x)^2} = \alpha \frac{(\Delta t)}{(\Delta y)^2} = 0.4$$

(a) Sketch a diagram for this model. (Hint: There should be symmetry in the nodes.)

[7 marks]

(b) Apply Taylor's Expansion to $\frac{\partial^2 T}{\partial x^2}$ of node 5, using finite central difference. (Reminder: x is along the horizontal direction.)

[2 marks]

Continued...

(c) Apply Taylor's Expansion to $\frac{\partial^2 T}{\partial y^2}$ of node 5, using finite central difference.
(Reminder: y is along the vertical direction.)

[2 marks]

(d) Apply Taylor's Expansion to $\frac{\partial T}{\partial t}$ of node 5, using finite forward difference.

[2 marks]

(e) Using the answers from parts (b), (c), and (e), simplify the temperature change equation for node 5 into an equation that includes the following variables:
$T_5^{t+1}, T_2^t, T_4^t, T_5^t, T_6^t, , T_8^t$
Convert any other variables into numbers.

[4 marks]

(f) Using the boundary conditions, convert the answer in part (e) to just T_5^t and T_5^{t+1}.
[2 marks]

(g) Find temperatures for T_5 at times $t = 0, 1, 2, 3$

[4 marks]

(h) From the answers in part (g), state whether the results have convergence or divergence.

[2 marks]

Continued...

Appendix

Explicit Euler method

$$y_{i+1} = y_i + hf(x_i, y_i)$$

Implicit Euler method

$$y_{i+1} = y_i + hf(x_{i+1}, y_{i+1})$$

Modified Euler method (predictor-corrector)

Step 1: $K_1 = f(x_i, y_i),\ y_{i+1}^P = y_i + hK_1$

Step 2: $K_2 = f(x_{i+1}, y_{i+1}^P),\ y_{i+1}^C = y_i + \frac{h}{2}(K_1 + K_2)$

Midpoint method

Step 1: $K_1 = f(x_i, y_i),\ y_m = y_i + \frac{h}{2}K_1$

Step 2: $K_2 = f(x_m, y_m),\ y_{i+1} = y_i + hK_2$

Fourth-order Runge-Kutta

Step 1: $K_1 = f(x_i, y_i),\ y_{m1} = y_i + \frac{h}{2}K_1$

Step 2: $K_2 = f(x_m, y_{m1}),\ y_{m2} = y_i + \frac{h}{2}K_2$

Step 3: $K_3 = f(x_m, y_{ym2}),\ y_{m3} = y_i + hK_3$

Step 4: $K_4 = f(x_{i+1}, y_{m3}),\ y_{i+1} = y_i + \frac{h}{6}(K_1 + 2K_2 + 2K_3 + K_4)$

Second-order Adams-Bashforth

$$y_{i+1} = y_i + \frac{h}{2}(3f_i - f_{i-1})$$

Third-order Adams-Bashforth

$$y_{i+1} = y_i + \frac{h}{12}(23f_i - 16f_{i-1} + 5f_{i-2})$$

Fourth-order Adams-Bashforth

$$y_{i+1} = y_i + \frac{h}{24}(55f_i - 59f_{i-1} + 37f_{i-2} - 9f_{i-3})$$

Second-order Adams-Moulton

$$y_{i+1} = y_i + \frac{h}{2}(f_i + f_{i+1})$$

Third-order Adams-Moulton

$$y_{i+1} = y_i + \frac{h}{12}(5f_{i+1} + 8f_i - f_{i-1})$$

Continued...

Fourth-order Adams-Moulton

$$y_{i+1} = y_i + \frac{h}{24}(9f_{i+1} + 19f_i - 5f_{i-1} + f_{i-2})$$

First derivative, forward difference

$$y' = \frac{1}{h}(y_{i+1} - y_i)$$

First derivative, backward difference

$$y' = \frac{1}{h}(y_i - y_{i-1})$$

First derivative, central difference

$$y' = \frac{1}{2h}(y_{i+1} - y_{i-1})$$

Second derivative, forward difference

$$y'' = \frac{1}{h^2}(y_i - 2y_{i+1} + y_{i+2})$$

Second derivative, backward difference

$$y'' = \frac{1}{h^2}(y_{i-2} - 2y_{i-1} + y_i)$$

Second derivative, central difference

$$y'' = \frac{1}{h^2}(y_{i-1} - 2y_i + y_{i+1})$$

Jacobi method

$$x_i^{k+1} = \frac{1}{a_{ii}}\left(b_i - \sum_{j=1}^{i-1} a_{ij}x_j^k - \sum_{j=i+1}^{n} a_{ij}x_j^k\right)$$

Gauss-Seidel method

$$x_i^{k+1} = \frac{1}{a_{ii}}\left(b_i - \sum_{j=1}^{i-1} a_{ij}x_j^{k+1} - \sum_{j=i+1}^{n} a_{ij}x_j^k\right)$$

Successive over-relaxation method

$$x_i^{k+1} = (1-\omega)x_i^k + \frac{\omega}{a_{ii}}\left(b_i - \sum_{j=1}^{i-1} a_{ij}x_j^{k+1} - \sum_{j=i+1}^{n} a_{ij}x_j^k\right)$$

Jacobian matrix

$$[J] = \begin{bmatrix} \frac{\partial f_i}{\partial x_1} & \frac{\partial f_i}{\partial x_2} & \cdots & \frac{\partial f_i}{\partial x_n} \\ \frac{\partial g_i}{\partial x_1} & \frac{\partial g_i}{\partial x_2} & \cdots & \frac{\partial g_i}{\partial x_n} \\ \frac{\partial h_i}{\partial x_1} & \frac{\partial h_i}{\partial x_2} & \cdots & \frac{\partial h_i}{\partial x_n} \\ \vdots & \vdots & \cdots & \vdots \end{bmatrix}$$

End of Paper